وزارة التعليم العالي والبحث العلمي
جامعة الكرخ للعلوم
كلية العلوم
قسم الفيزياء الطبية

ميكانيك I

المرحلة الأولى – الفصل الدراسي الأول

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قسم الفيزياء الطبية			كلية العلوم		جامعة الكرخ للعلوم	
المرحلة الاولى	لغة التدريس	عدد الوحدات	عدد الساعات العملي	عدد ساعات النظري	رمز المقرر	اسم المقرر
الفصل الدراسي الاول	English	3	2	2		Mechanics I

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References:

1- Halliday, Resnick and Walker; Fundamentals of Physics; 8th edition 2008.
2- Physics part I by Robert Resnick and David Halliday
3- Physics by Alenso and Finn 1981
4- University Physics by Francis and others 1982
5- Principle of physics by Jerry B. Marion and William F. Hornyak 1984
6- اساسيات الفيزياء بوش وجيرد

What is Physics?

The branch of science concerned with the nature and properties of matter and energy.

The subject matter of physics includes **mechanics**, **heat**, **light**, **radiation**, **sound**, **electricity**, **magnetism**, and the structure of **atoms** .. etc.

What is Mechanics?

Mechanics is an area of physical science concerned with the behaviour of physical bodies when subjected to forces or displacements, and the subsequent effects of the bodies on their environment.

Physical Units

The basic mechanical units are those of:

- **Mass (M)** **Kilogram**
- **Length (L)** **Meter**
- **Time (T)** **Second**

Above quantities is called **"Fundamental Quantities"**

Some other quantities called "**Derived Quantities**" which is derived from the fundamentals quantities such as:

- **Velocity** **m/s**
- **Acceleration** **m/s^2**
- **Force** **$Kg.m/s^2$ = Newton (N)**
- **Area (A)** **m^2**
- **Volume (V)** **m^3**

	SI Units (MKS)	(CGS)	U.S. Common
Length (L)	meter (m)	centimeter (cm)	foot (ft)
Time (T)	second (s)	second (s)	second (s)
Mass (M)	kilogram (kg)	gram (gm)	slug
Velocity (L/T)	m/s	cm/s	ft/s
Acceleration (L/T^2)	m/s^2	cm/s^2	ft/s^2
Force (ML/T^2)	kg m/s^2=Newton(N)	gm cm/s^2= dyne	slug ft/s^2=pound(lb)
Work (ML^2/T^2)	N m = joule (j)	dyne cm = erg	lb ft = ft lb
Energy (ML^2/T^2)	joule	erg	ft lb
Power (ML^2/T^3)	j/s = watt (W)	erg/s	ft lb/s

Note:

* System International of Units **(SI Units)**
* The centimetre–gram–second system of units **(CGS)**
* United States customary system **(U.S. Common)**

SI Units Prefixes

Factor	Name	Symbol
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

Factor	Name	Symbol
10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da

Examples:

- 1 A° (Angstrom) = 10^{-10} m.
- 1 cm = 10^{-2} m = 10^{-5} km.
- 1 m = 100 cm = 1000 mm
- 1 ton = 10^{6} gm = 10^{3} kg.
- 1 kg = 1000 gm.
- 1 Hour = 1 Hr = 60 min = 3600 sec.
- Density (ρ) = mass/volum = kg.m-3 (mass per unit volum)
- 10-6 kg = 1 mg (one milligram), Not that 10-6 kg = 1 μkg (one microkilogram)
- 169 000 mm = 16 900 cm = 169 m = 0.169 km

- **1day=24hours, 1hour=60minutes and 1 minute = 60 seconds**
- **Thus: 1 day 24× 60 ×60 = 86400 sec**

Error Measurement

The difference in the true value and measured value is called error.

There are two types of errors:

- Random error: caused by the person doing the experiment.
- Systematic error: due to the system or apparatus being used.

Mathematical description of errors:

Absolute error: It is the magnitude of difference between true value of quantity and the measurement value.
If p is the measured quantity then absolute error expressed as $\pm\Delta p$

Relative error: The ratio of absolute error to the true value of the physical quantity is called relative error.
Here $\pm\Delta p/p$ is the relative error.

Percentage error: relative error * 100% = $\pm\Delta p/p * 100\%$.

Scientific Hypothesis, Theory, or Law?

Hypothesis

A hypothesis is a reasonable guess based on what you know or observe, and hypotheses are proven and disproven all of the time.

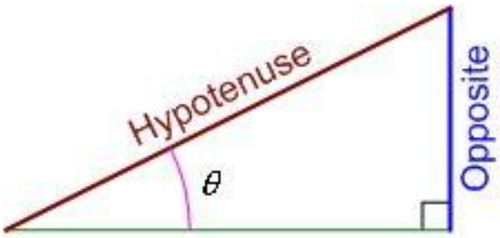
Theory

A scientific theory consists of one or more hypotheses that have been supported through repeated testing.

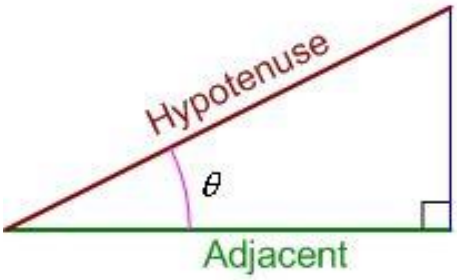
Law

Scientific laws are short, sweet, and always true. They're often expressed in a single statement and generally rely on a concise mathematical equation.

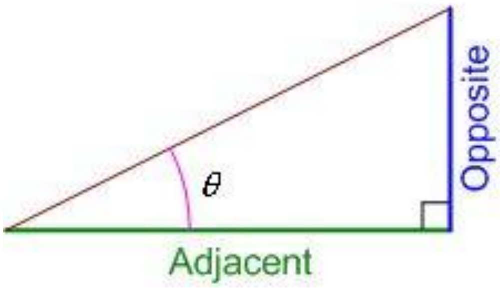
SIN COS and TAN



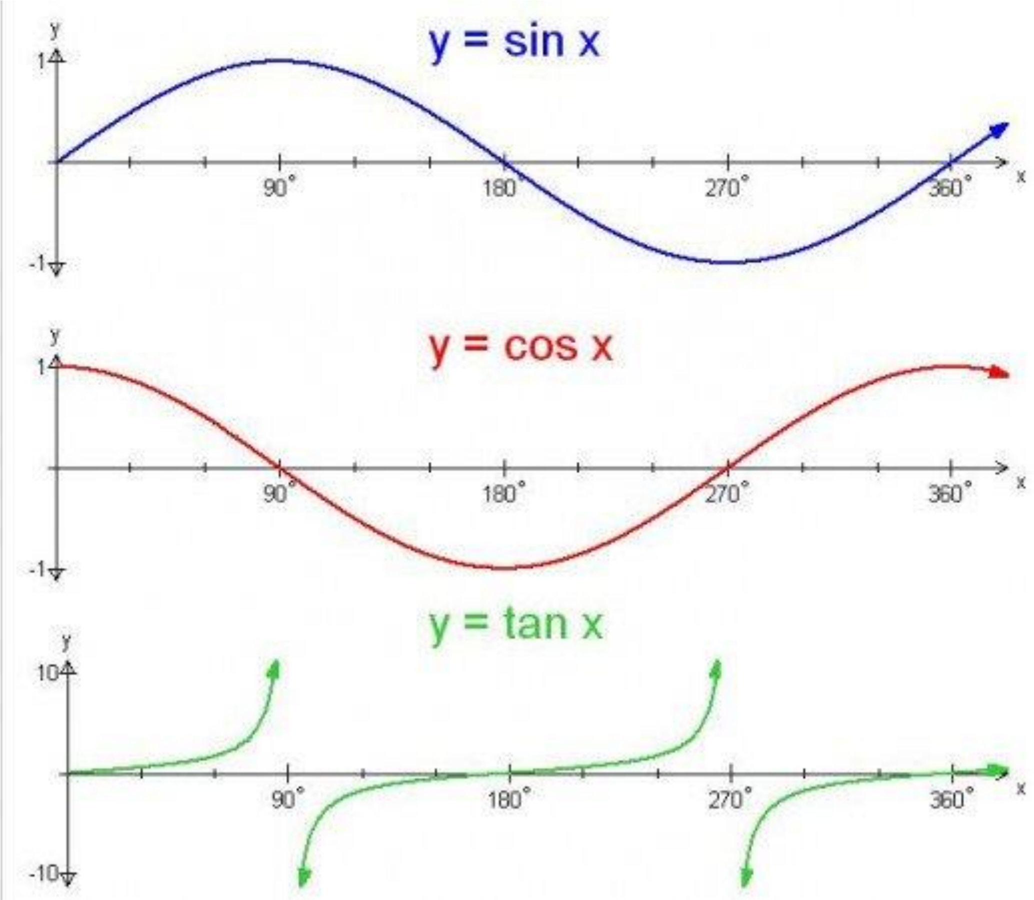
$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

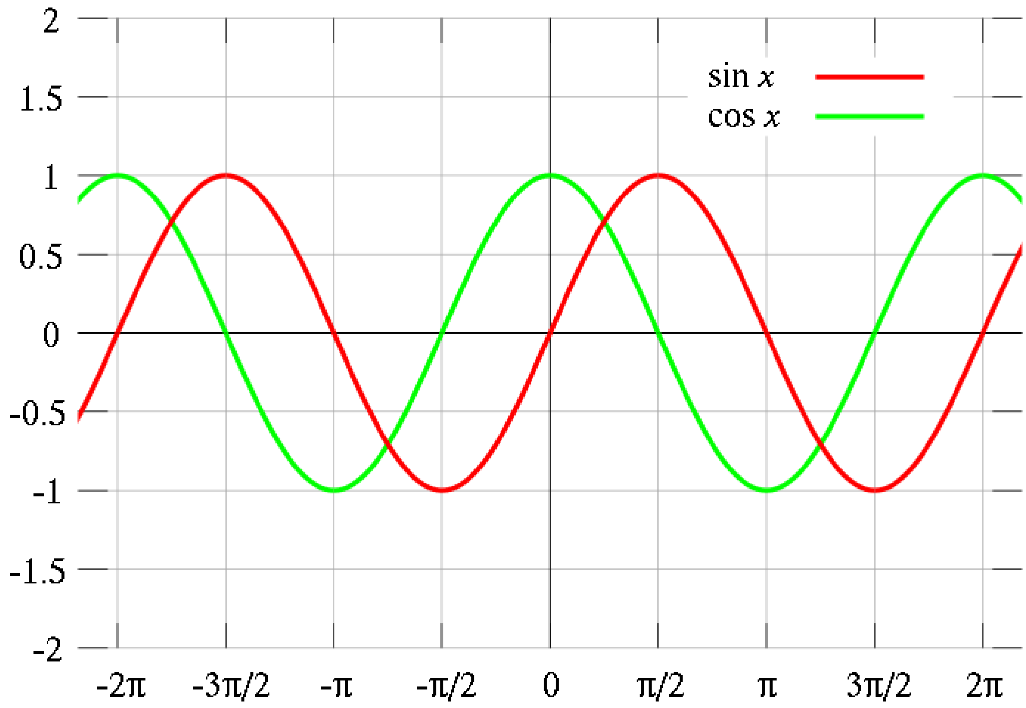


$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$



$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$


y = sin x
y = cos x
y = tan x
90°
180°
270°
360°


sin x
cos x
-2π
-3π/2
-π
-π/2
0
π/2
π
3π/2
2π

Greek Letters

GREEK ALPHABET

By Ben Crowder • bencrowder.net • Last modified 2 May 2012

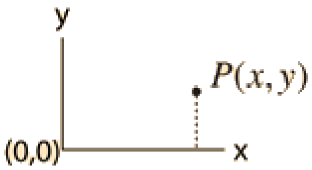
Αα **ALPHA** [a] *ἄλφα*	Ββ **BETA** [b] *βῆτα*	Γγ **GAMMA** [g] *γάμμα*	Δδ **DELTA** [d] *δέλτα*	Εε **EPSILON** [e] *ἒ ψιλόν*	Ζζ **ZETA** [dz] *ζῆτα*
Ηη **ETA** [ɛː] *ἦτα*	Θθ **THETA** [t^h] *θῆτα*	Ιι **IOTA** [i] *ἰῶτα*	Κκ **KAPPA** [k] *κάππα*	Λλ **LAMBDA** [l] *λάμβδα*	Μμ **MU** [m] *μῦ*
Νν **NU** [n] *νῦ*	Ξξ **XI** [ks] *ξεῖ*	Οο **OMICRON** [o] *ὂ μικρόν*	Ππ **PI** [p] *πεῖ*	Ρρ **RHO** [r] *ῥῶ*	Σσς **SIGMA** [s] *σῖγμα*
Ττ **TAU** [t] *ταῦ*	Υυ **UPSILON** [ʉ] *ὖ ψιλόν*	Φφ **PHI** [p^h] *φεῖ*	Χχ **CHI** [k^h] *χεῖ*	Ψψ **PSI** [ps] *ψεῖ*	Ωω **OMEGA** [ɔː] *ὦ μέγα*

Position

Specifying the position of an object is essential in describing motion. In one dimension some typical ways are

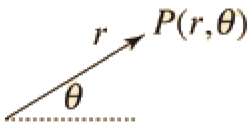
x(t) is used to represent position as a function of time.

In two dimensions, either **Cartesian** or **Polar Coordinates** may be used, and the use of unit vectors is common. A position vector r may be expressed in terms of the unit vectors.



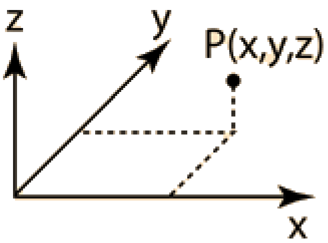
$$\vec{r} = x\vec{i} + y\vec{j} = r\vec{1}_r$$

$\vec{r}(t)$ describes position as a function of time



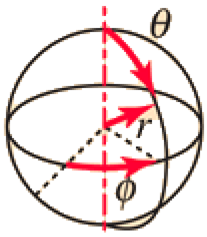
Where i and j are called unit vectors for x and y axes respectively. Unit vector is whose magnitude is one.

In three dimensions, **Cartesian** or **Spherical Polar Coordinates** are used.



In cartesian coordinates

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

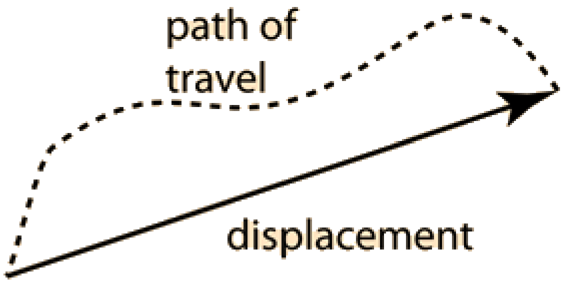


The vector change in position associated with a motion is called the displacement.

Displacement and Velocity

The displacement is the **vector distance** from some initial point to a final point.

The distance traveled divided by the time is called the **speed**, while the displacement divided by the time defines the average **velocity**.



Velocity is a vector quantity and the speed is a scalar quantity.

$$\textbf{Speed} = \frac{\textbf{Distance (m)}}{\textbf{Time (sec)}}$$ **(Scalar Quantity)**

$$\textbf{Velocity} = \frac{\textbf{Displacement(m)}}{\textbf{Time (sec)}}$$ **(Vector Quantity)**

For the special case of straight line motion in the x direction, the average velocity takes the form:

$$v_{average} = \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

The units for velocity can be implied from the definition to be **meters /second** or **(m/s).**

Acceleration

Acceleration is defined as the rate of change of velocity. Acceleration is inherently a vector quantity.

An object will have non-zero acceleration if its speed and/or direction is changing. The average acceleration is given by

$$\vec{a}_{average} = \bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Where the small arrows indicate the vector quantities.

The units for acceleration can be implied from the definition as **m/s^2**.

Description of Motion in One Dimension

Motion is described in terms of displacement (x), time (t), velocity (v), and acceleration (a).

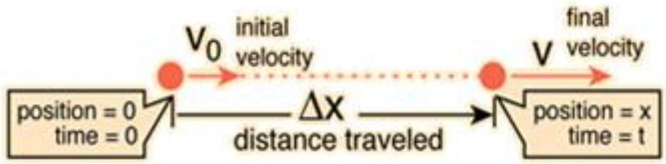
Velocity is the rate of change of displacement and the acceleration is the rate of change of velocity. The average velocity and average acceleration are defined by the relationships:

Average velocity: $\bar{v} = \frac{\Delta x}{\Delta t}$

Average acceleration: $\bar{a} = \frac{\Delta v}{\Delta t}$

Where the Greek letter Δ indicates the change in the quantity following it.

This can be related to a simple sketch for motion in the x direction:



With these choices, the above can be written: $\bar{V} = \frac{X}{t}$ or $X = \bar{V}\,t$

where both x and t are set equal to zero at the left hand point.

For the case of constant acceleration, the average velocity can also be expressed as:

$$\bar{V} = \frac{V_0 + V}{2}$$

When the acceleration is constant, the average acceleration and instantaneous acceleration are equal and we can write:

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_o}{t - 0}$$

Here v_o is the velocity at time t=0 and v is the velocity at any later time t. We can recast this equation as:

$$a = \frac{v - v_o}{t - 0}$$

$$a\,t = v - v_o$$

$$v = v_o + a\,t$$

Now,

$$x = \bar{v}\,t \dots\dots\dots (1)$$

But, $\bar{v} = \frac{v_o + v}{2}$

$$v = v_o + a\,t \dots\dots\dots (2)$$

By combining equations (1) and (2), lead to the useful form in equation (3) below.

$$x = \bar{v}\,t = \left[\frac{v_o + v}{2}\right] t$$

And substituting from equation (2) we get

$$x = \bar{v}\, t = \left[\frac{v_o + v_o + at}{2}\right] t$$

$$x = v_o t + \frac{1}{2} a t^2 \;\ldots\ldots\ldots (3)$$

Also, by combining equations (1) and (2), lead to the useful form in equation (4) below.

$$x = \bar{v}\, t \;\ldots\ldots\ldots (1)$$

But, $\bar{v} = \frac{v_o + v}{2}$

$$v = v_o + a\, t \;\ldots\ldots\ldots (2)$$

$$x = \bar{v}\, t = \left[\frac{v_o + v}{2}\right] t$$

And substituting from equation (2) we get

$$x = \left[\frac{v_o + v}{2}\right]\left[\frac{v_o - v}{a}\right]$$

$$x = \left[\frac{v^2 - v_o^2}{2a}\right]$$

$$v^2 = v_o^2 + 2ax \;\ldots\ldots\ldots (4)$$

If the acceleration is constant, then equations 1, 2 and 3 represent a complete description of the motion. Equation 4 is obtained by a combination of the others.

Free Fall Acceleration

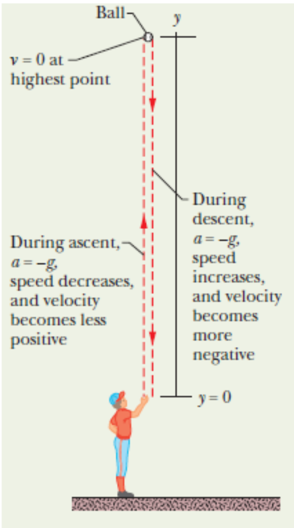
The free-fall acceleration near Earth's surface is $a = -g = -9.8\ m/s^2$, and the magnitude of the acceleration is g = 9.8 m/s2. Do not substitute -9.8 m/s2 for g.

The equations of motion for constant acceleration (that described praviously) also apply to free fall near Earth's surface; that is, they apply to an object in vertical flight, either up or down, when the effects of the air can be neglected.

<u>However, note that for free fall:</u>

(1) The directions of motion are now along a vertical y-axis instead of the x-xis, with the positive direction of y upward. (This is important for later chapters when combined horizontal and vertical motions are examined.)

(2) The free-fall acceleration is negative that is, downward on the y-axis, toward Earth's center and so it has the value -g in the equations.



<u>Problem 1</u>

A car accelerates from 25 m/s to 40 m/s in 9 seconds. Solve for the acceleration.

Sol.

V = Vo + at

40 m/s = 25 m/s + (9 s) a

15 m/s = (9 s) a

a = 1.67 m/s²

Vectors and Scalars

All quantities can by divided into two categories: scalars and vectors.

Scalars: Physical quantities that are fully described by a magnitude (or numerical value) alone. An example of scalars quantities are mass, volume, time and temperature.

Vectors: Physical quantities that are fully described by both a magnitude and a direction. An example of vectors quantities are displacement, force and velocity.

Consider the following example in the figure below the vector $\vec{A}$.

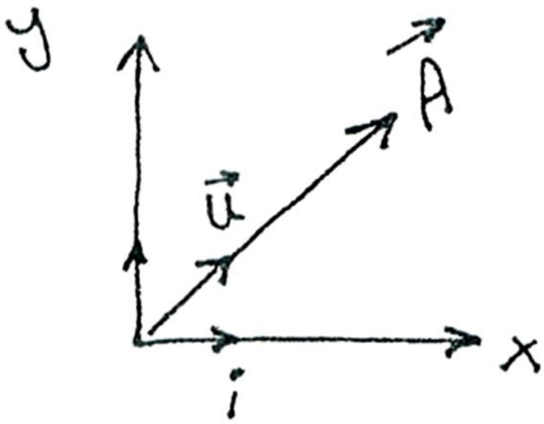
$|\vec{A}|$ is the magnitude of $\vec{A}$

$|\vec{A}| = \sqrt{x^2 + y^2}$

$\vec{A} = \vec{u}\,|\vec{A}|$ or $\vec{u} = \frac{\vec{A}}{|\vec{A}|}$

$\Rightarrow |\vec{u}| = 1$

Therefore any $|Unit\ vector| = 1$



If we have two points

$\boldsymbol{P} = (\boldsymbol{x_1}, \boldsymbol{y_1})$ **and** $\boldsymbol{Q} = (\boldsymbol{x_2}, \boldsymbol{y_2})$

The vector from P to Q is:

$$\overrightarrow{\boldsymbol{PQ}} = (\boldsymbol{x_2} - \boldsymbol{x_1}, \boldsymbol{y_2} - \boldsymbol{y_1})$$

Similar things in the 3D

For $\boldsymbol{P = (x_1, y_1, z_1)}$ **and** $\boldsymbol{Q = (x_2, y_2, z_2)}$

The vector from P to Q is:

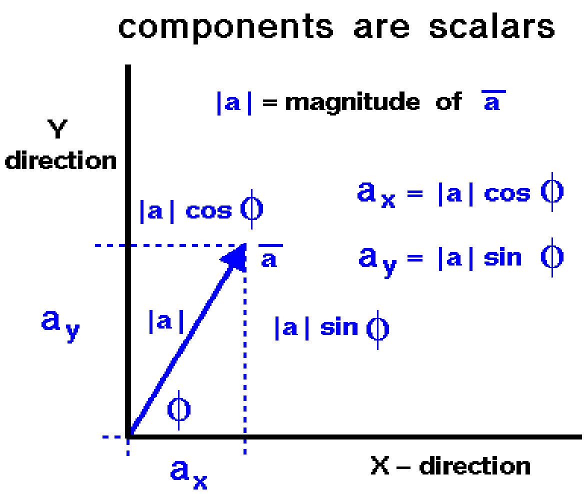
$$\overrightarrow{PQ} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

Thus, the vector is written in the form of: $\vec{A} = a_x i + a_y j + a_z k$

Component of a vector

Any vector $\bar{A}$ in the xy-plane can be written as a sum of two other vectors, one along the x-axis (a_x) and the other along the y-axis (a_y). These are called the components of $\bar{A}$

$$a_x = |a| \cos \emptyset \qquad a_y = |a| \sin \emptyset$$



The magnitude is, $$|a| = \sqrt{{a_x}^2 + {a_y}^2}$$

$$\bar{a} = |\bar{a}|\, \bar{u} \;\Rightarrow\; \bar{u} = \frac{\bar{a}}{|\bar{a}|}$$

Where $\bar{u}$ is called unit vector, and its absolute vale is always equal to one.

$$|\bar{u}| = 1$$

(In plane x-y) the vector components are

$\vec{V} = \overrightarrow{V_x} + \overrightarrow{V_y}$ (In plane x-y)

$V_x = V\cos\alpha$

$V_y = V\sin\alpha$

$\overrightarrow{V_x} = i\,V_x$ and $\overrightarrow{V_y} = j\,V_y$

$\vec{V} = i\,V_x + j\,V_y$

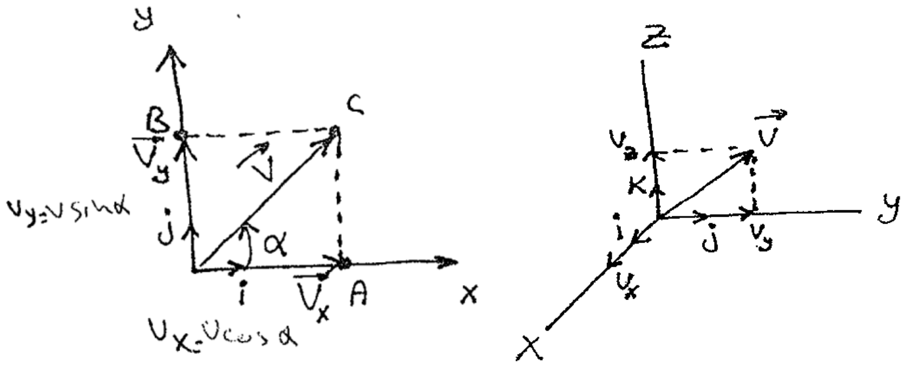
Similarly in the case of three rectangular components in space (x, y, z)

$$\vec{V} = i\,V_x + j\,V_y + k\,V_z$$

k: unit vector for z- axis

The magnitude of vector in space is:

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2}$$



The angles between vectors and x, y, z axis is α, β, γ

$$\vec{V} = i\,V_x + j\,V_y + k\,V_z$$

$$V_x = V\cos\alpha\,, V_y = V\cos\beta\,, V_z = V\cos\gamma$$

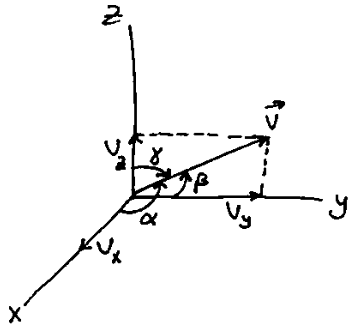
$$\vec{V} = V(\cos\alpha\ i + \cos\beta\ j + \cos\gamma\ k)$$

$$\vec{u} = \frac{\vec{V}}{V} = \cos\alpha\ i + \cos\beta\ j + \cos\gamma\ k$$

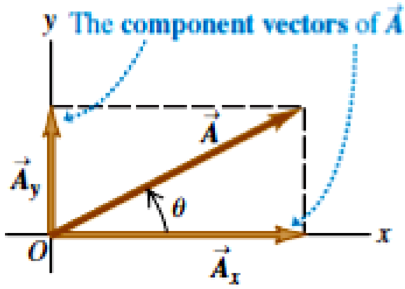
As we know the unit vector is $|\vec{u}| = 1$

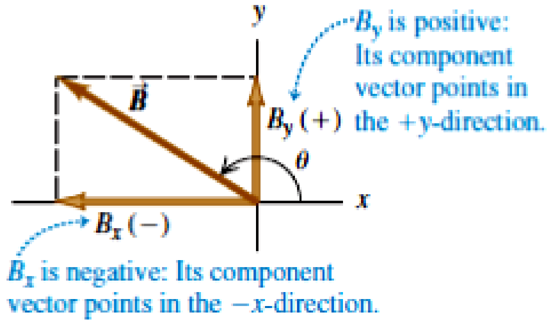
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

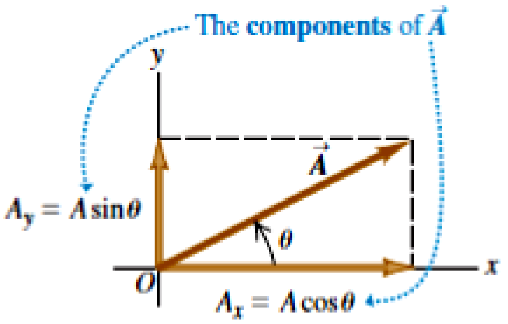
This the relation between angles.

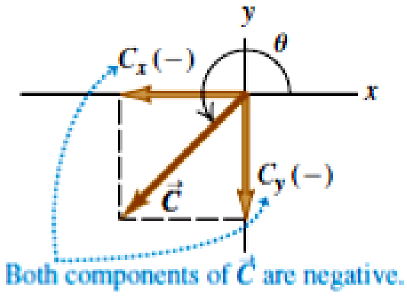


Figures below represent vectors in terms of component vectors. The components of a vector may be positive or negative numbers.







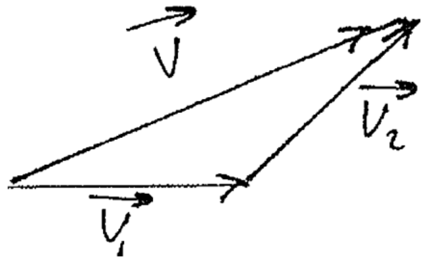


Vector Operations

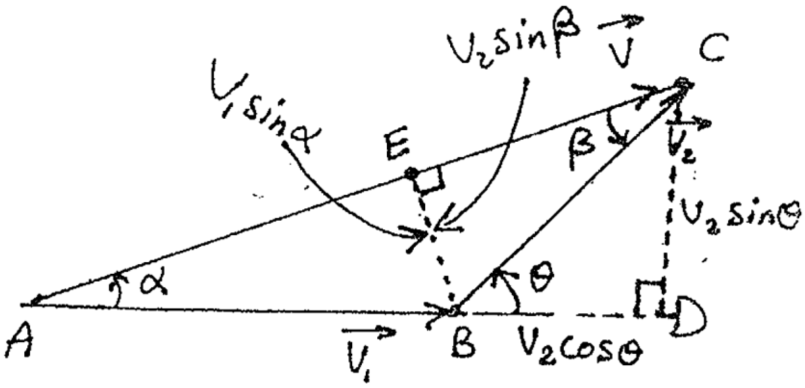
Both a magnitude and a direction must be specified for a vector quantity. Any number of vector quantities of the same type (i.e., same units) can be combined by basic vector operations.

Graphical Vector Addition

Adding two vectors V_1 and V_2 graphically can be visualized like two successive walks, with the vector sum being the vector distance from the beginning to the end point. Representing the vectors by arrows drawn to scale, the beginning of vector V_2 is placed at the end of vector V_1. The vector sum V can be drawn as the vector from the beginning to the end point.



$$\vec{V} = \vec{V_1} + \vec{V_2}$$



From above figure,

Where θ is the angle between $\vec{V_1}$ and $\vec{V_2}$.

$$(AC)^2 = (AD)^2 + (DC)^2$$

But $AD = AB + BD = V_1 + V_2 \cos\theta$

And $DC = V_2 \sin\theta$

Therefore $V^2 = (V_1 + V_2 \cos\theta)^2 + (V_2 \sin\theta)^2$

$$V^2 = V_1^2 + V_2^2 \cos^2\theta + 2V_1V_2\cos\theta + V_2^2\sin^2\theta$$

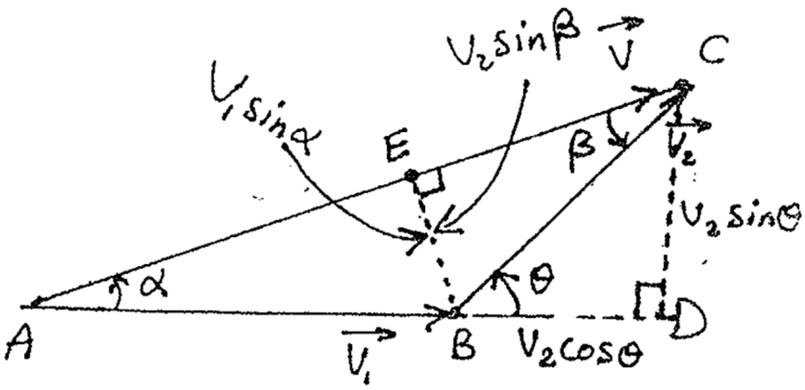
$$V^2 = V_1^2 + V_2^2(\cos^2\theta + \sin^2\theta) + 2V_1V_2\cos\theta$$

$$V^2 = V_1^2 + V_2^2 + 2V_1V_2\cos\theta$$

$$\Rightarrow V = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos\theta}$$

Above equation represent the magnitude of V.

To determine the direction of $\vec{V}$ we need to know the angles α or β where:



From above figure,

$CD = AC \sin\alpha$ **(From triangle ACD)**

$CD = BC \sin\theta$ **(From triangle BDC)**

But, $AC = V, \quad BC = V_2$

$$\Rightarrow V \sin\alpha = V_2 \sin\theta \quad or \quad \frac{V}{\sin\theta} = \frac{V_2}{\sin\alpha}$$

Similarly, $BE = V_1 \sin\alpha = V_2 \sin\beta$

$$\Rightarrow \frac{V_2}{\sin\alpha} = \frac{V_1}{\sin\beta}$$

By combining both results, we get the symmetrical relation:

$$\frac{V}{\sin\theta} = \frac{V_1}{\sin\beta} = \frac{V_2}{\sin\alpha}$$

$$\tan\alpha = \frac{V_2}{V_1}$$

The difference between vectors:

$$\vec{D} = \vec{V_1} - \vec{V_2} = \vec{V_1} + (-\vec{V_2})$$

$$\pi - \theta \;\; is\ the\ angle\ between\ vectors\ \vec{V_1} and\ (-\vec{V_2})$$

The magnitude of the difference is:

$$D = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos(\pi-\theta)}$$

Since, $\cos(\pi-\theta) = \cos\pi\,.\cos\theta - \sin\pi\,.\,\sin\theta = -\cos\theta$

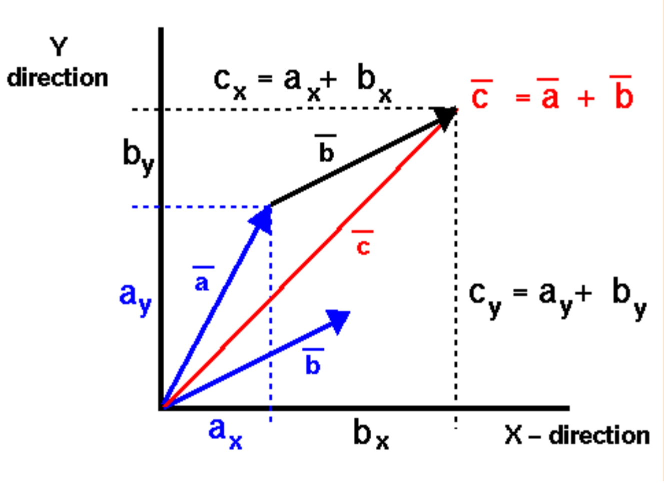
$$\Rightarrow D = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}$$

To add vectors in components:

The process can be done mathematically by finding the components of $\vec{a}$ and $\vec{b}$, combining to form the components of $\vec{c}$. as we see in the figure below.

$$\vec{c} = c_x + c_y$$

Where, $\quad c_x = a_x + b_x \quad$ and $\quad c_y = a_y + b_y$



Notes:

- The vector addition obeys the **commutative law.** i.e.

$$\bar{A} + \bar{B} = \bar{B} + \bar{A}$$

- The vector addition obeys the **associative law.** i.e.

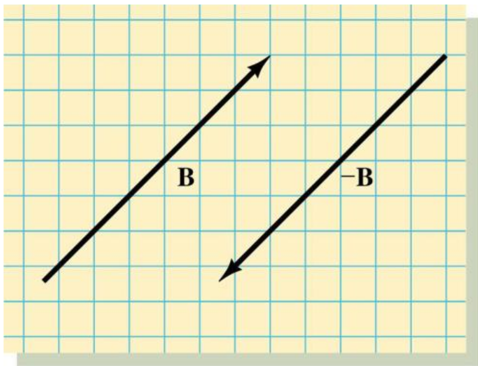
$$(\bar{A} + \bar{B}) + C = \bar{A} + (\bar{B} + \bar{C})$$

- Vector Subtraction

$$\bar{A} - \bar{B} = \bar{A} + (-\bar{B})$$

Where the vector $(-\bar{B})$ is the negative vector of $\bar{B}$.

$$\bar{B} + (-\bar{B}) = 0$$

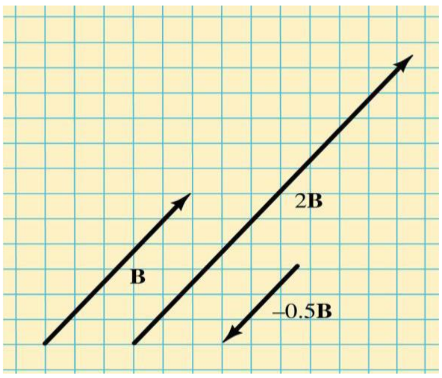


Multiplication of Vectors

Multiplication of a vector $\bar{B}$ by a scalar value b; determine bB

- The magnitude $|b\bar{B}| = |b||\bar{B}|$
- The direction of $b\bar{B}$ depends on the algebraic sign of b.

If **b > 0** then $b\bar{B}$ has the **same** direction as $\bar{B}$.
If **b < 0** then $b\bar{B}$ has the **opposite** direction of $\bar{B}$.



Scalar Product (Dot Product)

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

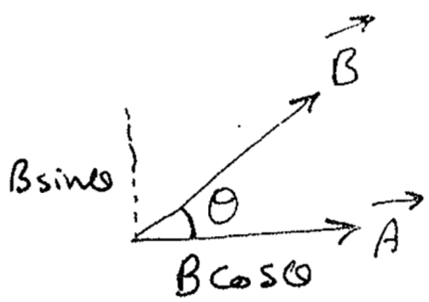
θ is the range between two vectors

$\vec{A} \cdot \vec{B} = 0$ **if the** $(\vec{A} \perp \vec{B})$ $\theta = 90$ **cos 90 = 0**

i.i = j.j = k.k = 1 **and** **i.j = j.k = k.i = 0**

also, $\vec{\boldsymbol{A}}\,.\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}\,.\vec{\boldsymbol{A}}$

$\vec{\boldsymbol{C}}\,.(\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}) = (\vec{\boldsymbol{C}}\,.\vec{\boldsymbol{A}}) + (\vec{\boldsymbol{C}}\,.\vec{\boldsymbol{B}})$

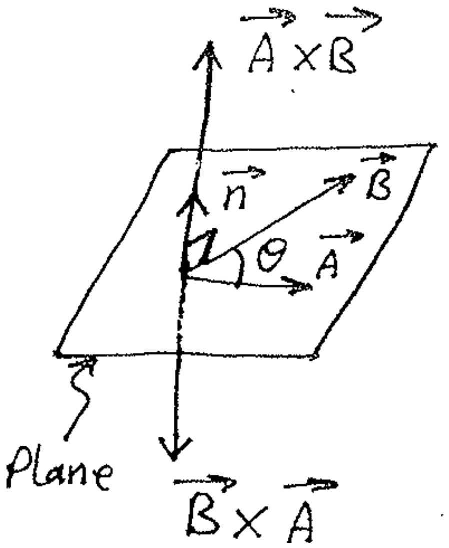


Vector Product (Cross Product)

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta\ \vec{n}$

θ is the range between two vectors.

$\vec{n}$ is the unit vector perpendicular on both $\vec{A}$ *and* $\vec{B}$ also on plane

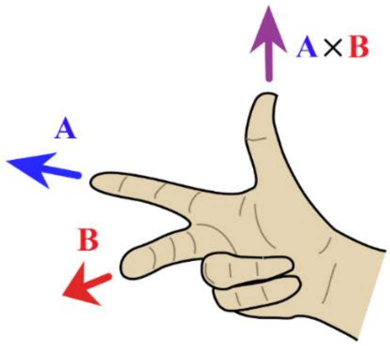


$\vec{\boldsymbol{N}} = \vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$ **Normal vector on plane contain $\vec{A}$ *and* $\vec{B}$**

$\vec{N} = \vec{n}|\vec{N}|$

$$\vec{n} = \frac{\vec{N}}{|\vec{N}|}$$

The direction of the new vector is determined by the right hand rule.



Notes:

1) Lets $\vec{A} = A_x i + A_y j + A_z k$ and $\vec{B} = B_x i + B_y j + B_z k$

$$\Rightarrow \vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= i(A_y B_z - A_z B_y) - j(A_x B_z - A_z B_x) + k(A_x B_y - A_y B_x)$$

2) If $\vec{A} \parallel \vec{B} \Rightarrow \vec{A} \times \vec{B} = 0 \Rightarrow \theta = 0 \quad sin\, 0 = 0$

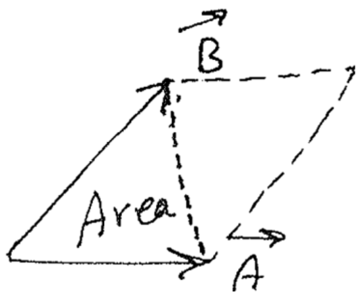
If $\vec{A} \perp \vec{B} \Rightarrow \vec{A} \times \vec{B} = AB \Rightarrow \theta = 90° \quad sin\, 90 = 1$

3) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

4) $i \times i = j \times j = k \times k = 0 \quad and \quad i \times j = k, \; j \times k = i, \; k \times i = j$

5) The area of triangle, by vector is:

$$Area = \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$$



Homework

Find the constant a, if the vector $\vec{A}$= 3i – j + 2k and the vector $\vec{B}$= a i + 2j – k are perpendicular.

Problem

What are the x and y components of a vector of magnitude of 8 and at angle 60° from the origin?

Sol.

Plot the components..

$$\cos 60 = \frac{x}{8}$$

X=4

$$\sin 60 = \frac{y}{8}$$

Y=$4\sqrt{3}$

Problem

Fine the vector $\overrightarrow{AB}$ from two points A (2,1) and B (-1,2), also find the unit vector in direction of $\overrightarrow{AB}$.

Sol.

$$\overrightarrow{AB} = (-1-2)i + (2-1)j = -3i + j$$

Unit vector $\vec{u} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{-3i+j}{\sqrt{(-3)^2+(1)^2}} = \frac{1}{\sqrt{10}}(-3i + j)$

<u>Home Work</u>

Given the two displacements

$$\vec{D} = 6\,i + 3\,j - 1\,k$$

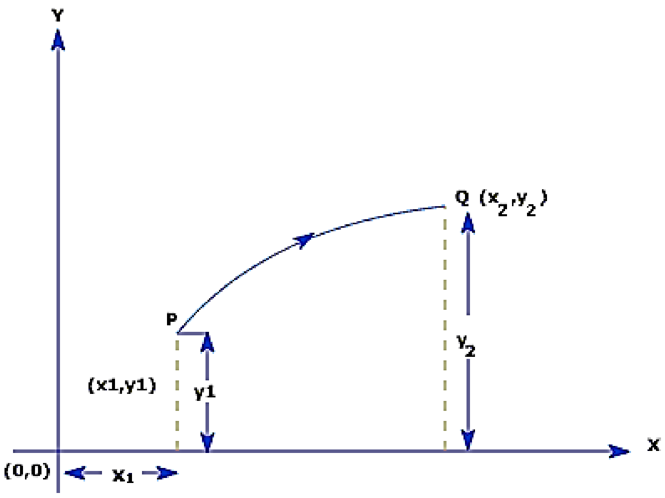
$$\vec{E} = 4\,i - 5\,j + 8\,k$$

Find the magnitude of the displacement $2\,\vec{D} - \vec{E}$.

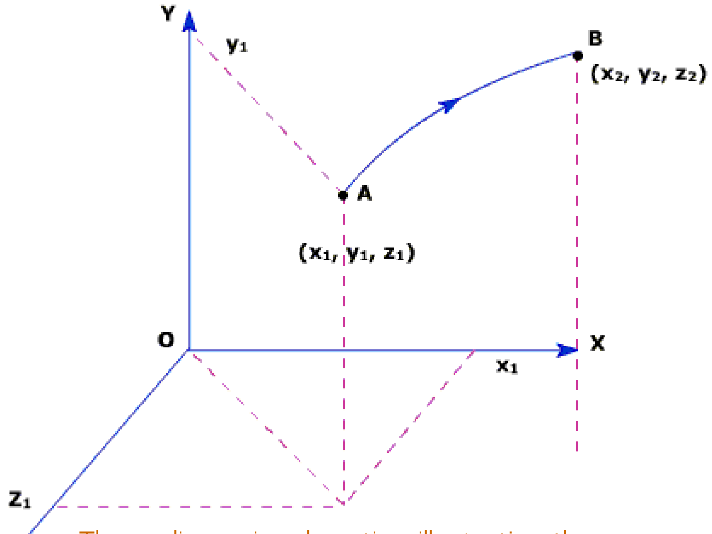
Motion in Two and Three Dimensions

As we learn in the first chapter, Motion in one dimension is also known as linear motion, a particle moving along a straight line is said to undergo one dimensional motion. In such a case, only one of the three rectangular coordinates changes with time. Examples of one dimensional motion are: Motion of a train along a straight line and an object, like a ball, falling freely, vertically under gravity.

A particle moving along a curved path in a plane has two dimensional motion. The figure below, illustrates a two dimensional motion, where a particle moves from P (x_1, y_1) to Q (x_2, y_2) along a curved path. Examples of two dimensional motion are: a satellite revolving round the Earth and projectile motion.



A particle moving in space has three dimensional motion. In this type of motion, all the three rectangular coordinates change with time. The figure below illustrates this type of motion where the particle moves from A to B and the corresponding rectangular coordinates change from (x1, y1, z1) to (x2, y2, z2). Examples of three dimensional motion are: A bird flying in the air and A kite flying in the air.



Three dimensional motion illustrating the position coordinates of A (x, y, z)

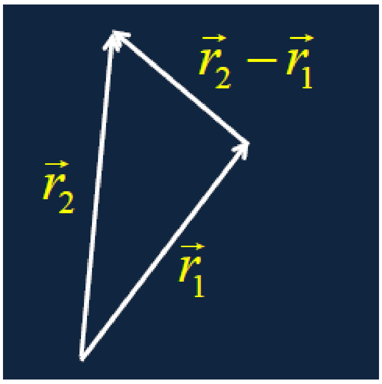
Average Velocity in Two Dimensions

Rate of change of position is velocity.

$$\text{Average velocity} = \frac{Displacement}{Time}$$

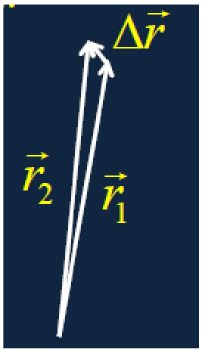
In moving from point $\vec{r_1}$ to $\vec{r_2}$, the average velocity is in the direction $\vec{r_2} - \vec{r_1}$:

$$\vec{v} = \frac{r_2 - r_1}{t_2 - t_1}$$



Instantaneous Velocity in Two Dimensions

Defined as the average velocity over a vanishingly small time interval: points in direction of motion at that instant.



The limit of average velocity as the time interval between measurements goes to zero:

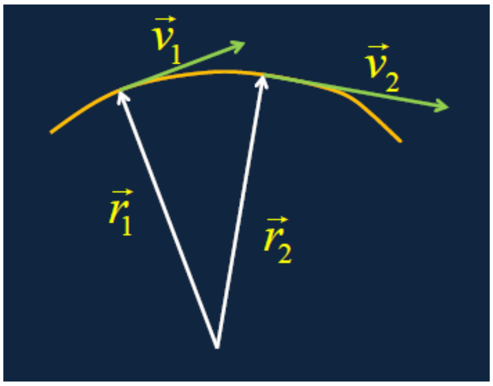
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

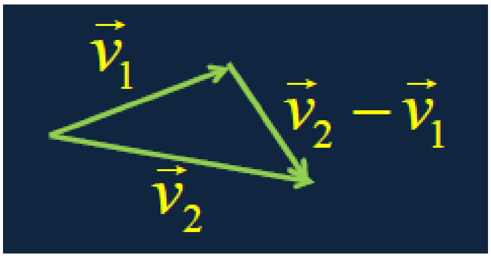
Note: $\Delta\vec{r}$ is small but that doesn't mean $\vec{v}$ has to be small, Δt is small too.

Average Acceleration in Two Dimensions

To simplify the acceleration in two dimensions will take the following example:

Assume a Car moving along curving road:

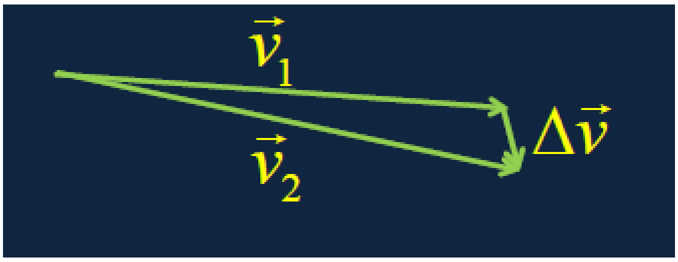




$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Note that the velocity vectors tails must be together to find the difference between them.

Instantaneous Acceleration in Two Dimensions



$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

Writing $\vec{a} = (a_x\,,\ \vec{a}_y)$ *and* $\vec{r} = (x, y)$

$$a_x = \frac{d^2\vec{x}}{dt^2} \quad and \quad a_y = \frac{d^2\vec{y}}{dt^2}$$

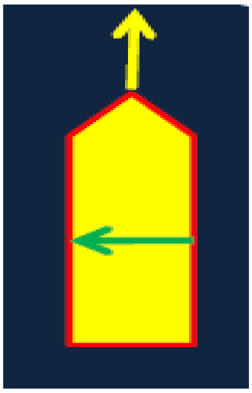
Note: as you would expect from the one-dimensional case.

Relative velocity

In general, when two observers measure the velocity of a moving body, they get different results if one observer is moving relative to the other. The velocity seen by a particular observer is called the velocity relative to that observer, or simply relative velocity.

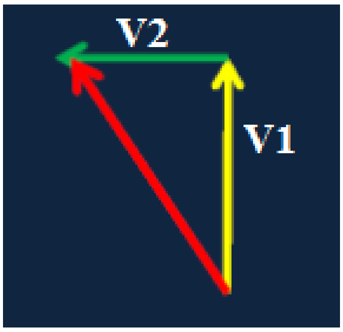
To understand the relative velocity will take the following example:

- A cruise ship is going north at 4 m/s through still water.
- You jog at 3 m/s directly across the ship from one side to the other.



In this case, what is your velocity relative to the water?

If the ship's velocity relative to the water is $\overrightarrow{v_1}$ and your velocity relative to the ship is $\overrightarrow{v_2}$ then your velocity relative to the water is:



$$\overrightarrow{v_1} + \overrightarrow{v_2}$$

Problem

The position of an electron is given by r = 3ti − 4t^{2}j + 2k (where t is in seconds and the coefficients have the proper units for r to be in meters). (a) What is v(t) for the electron? (b) In unit–vector notation, what is v at t = 2 s? (c) What are the magnitude and direction of v just then?

Sol.

(a) The velocity vector v is the time–derivative of the position vector r:

$$v = \frac{dr}{dt} = \frac{d}{dt}(3ti - 4t^2 j + 2k)$$

$$v = 3i - 8tj$$

where we mean that when t is in seconds, v is given in m/s.

(b) At t = 2 s, the value of v is:

$$\boldsymbol{v(t = 2\ s) = 3i - (8)(2)j}$$

$$\boldsymbol{v(t = 2\ s) = 3i - 16j \quad {}^{m}/_{s}}$$

(c) Using our answer from (b), at t = 2 s the magnitude of v is:

$$\boldsymbol{v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(3)^2 + (-16)^2 + (0)^2} = 16\ {}^{m}/_{s}}$$

Note that the velocity vector lies in the x-y plane (even though this is a three dimensional problem!) so that we can express its direction with a single angle, the usual angle measured anti-clockwise in the x-y plane from the x axis.

For this angle, we get:

$$\boldsymbol{\tan\theta = \frac{v_y}{v_x} = \frac{(-16)}{3} = 5.33}$$

$$\boldsymbol{\theta = \tan^{-1}(-5.33) = -79°}$$

Projectile Motion

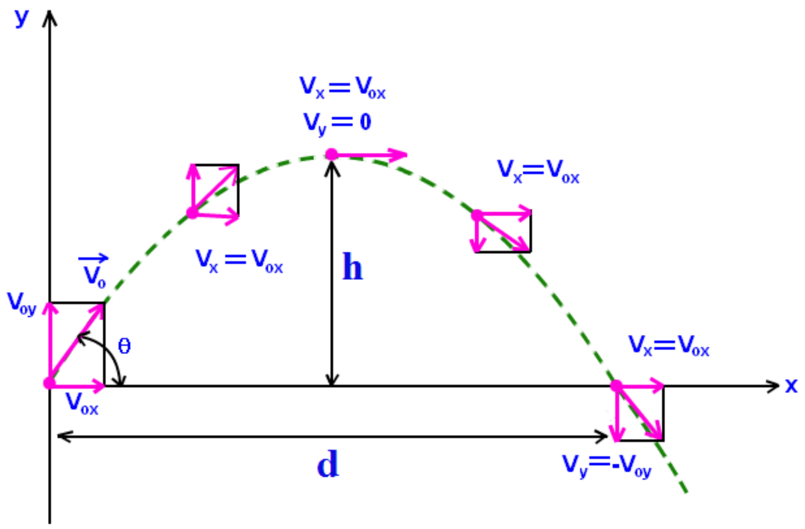
The name projectiles does not necessarily simply that you're dealing with some military device. It's just a general term, to describe anything moving freely through the air.

When a body is given an initial velocity and if later it moves purely under the influence of gravity, then the motion is called projectile motion.

- We will ignore two aspects of "real life" projectile motions (for simplicity):
  – Air resistance – Curvature of earth

- Key to solving projectile motion problem is to separate its x and y components and treat them separately

– Horizontal motion is with constant velocity
– Vertical motion is with constant gravitational acceleration



- X direction constant velocity
- Y direction constant acceleration due to gravity (-g)

$$v_{ox} = v_0 cos\theta$$

$$v_{oy} = v_0 sin\theta$$

The velocity at any time is given by:

$$v_x = v_{ox}$$

$$v_y = v_{oy} - gt$$

There is only acceleration in the vertical direction,

The velocity in the horizontal direction is constant, being equal to $v_0 cos\theta$.

The vertical motion of the projectile is the motion of a particle during its free fall. Here the acceleration is constant, being equal to **g.** The components of the acceleration are:

$$a_x = 0 \qquad \text{and} \qquad a_y = -g$$

The horizontal component of the velocity of the object remains unchanged throughout the motion.

The downward vertical component of the velocity increases linearly, because the acceleration due to gravity is constant. The accelerations in the x and y directions can be integrated to solve for the components of velocity at any time t, as follows:

$$v_x = v_0 cos(\theta)$$

$$v_y = v_0 sin(\theta) - gt$$

The magnitude of the velocity (under the Pythagorean theorem, also known as the triangle law):

$$v = \sqrt{v_x^2 + v_y^2}$$

At any time t, the projectile's horizontal and vertical displacement are:

$$x = v_0 t\, cos(\theta) \quad \dots\dots\dots (1)$$

$$y = v_0 t\, sin(\theta) - \frac{1}{2} g t^2 \quad \dots\dots\dots (2)$$

The magnitude of the displacement is:

$$r = \sqrt{x^2 + y^2}$$

From equations 1 and 2, If *t* is eliminated between these two equations the following equation is obtained:

$$\boldsymbol{y = \tan\theta . x - \frac{g}{2v_o^2 cos^2\theta} . x^2}$$

Since **g**, $\boldsymbol{\theta}$ and $\boldsymbol{v_0}$ are constants, the above equation is of the form:

$$\boldsymbol{y = ax - bx^2}$$

In which $\boldsymbol{a}$ and $\boldsymbol{b}$ are constants. This is the equation of a parabola, so the path is parabolic. The axis of the parabola is vertical.

If the projectile's position (x,y) and launch angle (θ) are known, the initial velocity can be found solving for $\boldsymbol{v_0}$ in the aforementioned parabolic equation:

$$\boldsymbol{v_o = \sqrt{\frac{x^2 g}{x \sin 2\theta - 2y\, cos^2\theta}}}$$

Time of flight or total time of the whole journey

The total time t for which the projectile remains in the air is called the time of flight.

$$y = v_0 t\, sin(\theta) - \frac{1}{2} g t^2$$

After the flight, the projectile returns to the horizontal axis (x-axis), so y=0

$$0 = v_0 t\, sin(\theta) - \frac{1}{2} g t^2$$

$$v_0 t\, sin(\theta) = \frac{1}{2} g t^2$$

$$v_0\, sin(\theta) = \frac{1}{2} g t$$

$$t = \frac{2\, v_0\, sin(\theta)}{g}$$

Maximum height of Projectile

The greatest height that the object will reach is known as the peak of the object's motion. The increase in height will last until $\boldsymbol{v_y = 0}$**,** that is:

$$v_y = v_0 sin(\theta) - gt$$

$$0 = v_0 sin(\theta) - gt_h$$

Time to reach the maximum height (h): Hence, the time is half of the time of flight

$$t_h = \frac{v_0\, sin(\theta)}{g}$$

From the vertical displacement of the maximum height of projectile:

$$h = v_o t_h \sin(\theta) - \frac{1}{2}\, g t_h^2 \qquad \Rightarrow h = \frac{v_o^2 sin^2(\theta)}{2g}$$

Maximum distance of Projectile

It is important to note that the range and the maximum height of the projectile does not depend upon its mass. Hence range and maximum height are equal for all bodies that are thrown with the same velocity and direction.

The horizontal range **d** of the projectile is the horizontal distance it has traveled when it returns to its initial height (y = 0).

$$y = v_0 t\ sin(\theta) - \frac{1}{2} g t^2$$

$$0 = v_0 t_d\ sin(\theta) - \frac{1}{2} g t_d^2$$

Time to reach ground: $$t_d = \frac{2\, v_0\, sin(\theta)}{g}$$

From the horizontal displacement the maximum distance of projectile:

$$d = v_o t_d \cos(\theta)$$

So, $$d = \frac{v_o^2}{g}\, sin(2\theta)$$

Note that **d** has its maximum value when:

$$sin\ (2\theta) = 1$$

$$2\theta = 90$$

$$\theta = 45°$$

Problem

A football is kicked with an initial velocity of 25 m/s at an angle of 45° with the horizontal. Determine the time of flight, the horizontal displacement, and the peak height of the football.

Sol.

$$t = \frac{2\ v_0\ sin(\theta)}{g}$$

$$t = \frac{2 * 25 * sin\ 45}{9.8} = 3.61\ sec$$

$$x = v_0 t\ cos(\theta)$$

$$x = 25 * 3.61 * cos\ 45 = 63.82\ m$$

$$h = \frac{v_o^2 sin^2(\theta)}{2g}$$

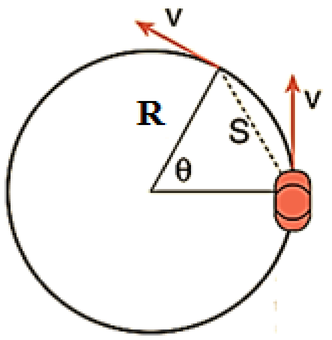
$$h = \frac{25^2 sin^2(45)}{2 * 9.8} = 15.9\ m$$

$$\mathrm{v_{ix}} = 16.5\ \mathrm{m/s}$$

Uniform Circular Motion: Angular Velocity

When a particle is moving in a circular path (or part of one) at constant speed, we say that the particle is in uniform circular motion. Even though the speed is not changing, the particle is accelerating because its velocity v is changing direction.

For circular motion at a constant speed v, the centripetal acceleration of the motion can be derived:



The velocity v being tangent to the circle and perpendicular to the radius (R).

$$S = R\,\theta$$

$$v = \frac{ds}{dt} = \frac{d}{dt}(R\theta) = \theta\frac{dR}{dt} + R\frac{d\theta}{dt}$$

$$v = R\frac{d\theta}{dt}$$

But, the Angular velocity is:

$$\omega = \frac{d\theta}{dt} \quad (Rad/sec)$$

$$\Rightarrow v = \omega\, R$$

Direction of ω is perpendicular to the plane of motion.

Note $\omega = constant$ in uniform circular motion, the motion is periodic and the particle is passes through each point of the circle at regular interval of time.

Let T the period, the time required for a complete turn or revolution, and the frequency ϑ is the number of revolution per unit time.

If the particle makes n revolution in time t:

$$The\ period \quad T = \frac{t}{n} \quad sec$$

$$The\ Frequency \quad \vartheta = \frac{n}{t} = \frac{1}{T} \quad Hz\ or\ sec^{-1}$$

rps = revolution per second = Hz

rpm = revolution per minute = $minute^{-1} = \frac{1}{60}\ Hz$

If ω is constant: $\int_{\theta_o}^{\theta} d\theta = \int_{t_o}^{t} \omega\ dt = \omega \int_{t_o}^{t} dt$

Or $\theta = \theta_o + \omega(t - t_o)$

If $\theta_o = 0\ and\ t_o = 0 \quad \Rightarrow \theta = \omega\ t \quad or \quad \omega = \frac{\theta}{t}$

And for complete revolution, $t = T \quad and \quad \theta = 2\pi$

$$\Rightarrow \omega = \frac{2\pi}{T} = 2\pi\ \vartheta$$

Circular Motion: Angular Acceleration

Angular acceleration:

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2} \qquad \left(\frac{rad}{sec^2}\right)$$

α is constant when the circular motion is uniformly accelerated.

$$\int_{\omega_o}^{\omega} d\omega = \int_{t_o}^{t} \alpha\ dt = \alpha \int_{t_o}^{t} dt$$

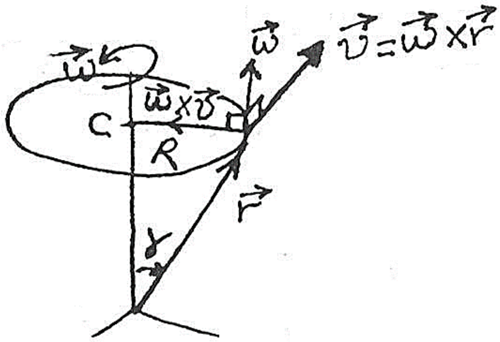
Or $\omega = \omega_o + \alpha(t - t_o)$

But, $\omega = \frac{d\theta}{dt} = \omega_o + \alpha(t - t_o) \Rightarrow \int_{\theta_o}^{\theta} d\theta = \int_{t_o}^{t} \omega_o dt + \alpha \int_{t_o}^{t} (t - t_o)\ dt$

$$\Rightarrow \theta = \theta_o + \omega_o(t - t_o) + \frac{1}{2}\,\alpha\,(t - t_o)^2$$

When $t_o = 0$ and $\theta_o = 0$ $\qquad \Rightarrow \theta = \omega_o t + \frac{1}{2}\,\alpha\,t^2$

Velocity and Acceleration in Circular Motion



$$R = r\sin\gamma$$

But, $\qquad v = \omega\,R$

$$\therefore v = \omega\,r\sin\gamma$$

$$\Rightarrow \vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{\omega} \times \frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r}$$

But $\vec{\omega}$ $is\ constant.$

$$\Rightarrow \vec{a} = \vec{\omega} \times \frac{d\vec{r}}{dt} \qquad \Rightarrow \vec{a} = \vec{\omega} \times \vec{v}$$

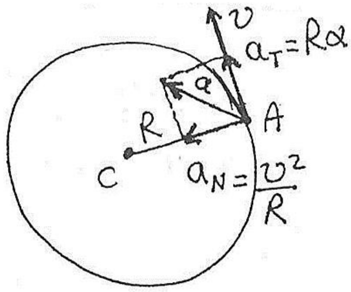
But $\vec{v} = \vec{\omega} \times \vec{r}$ $\qquad \Rightarrow \therefore\ \vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Note: In uniform circular motion, the acceleration is perpendicular to the velocity and points radially inward.

The Centripetal Acceleration:

The Centripetal Acceleration is the rate of change of tangential velocity. The word “centripetal” means towards the center.

Consider an object moving in a circle of radius R with constant angular velocity. The tangential speed is constant, but the direction of the tangential velocity vector changes as the object rotates.



$$a_N = \frac{v^2}{R} = \frac{(R\,\omega)^2}{R} = \omega^2\,R$$

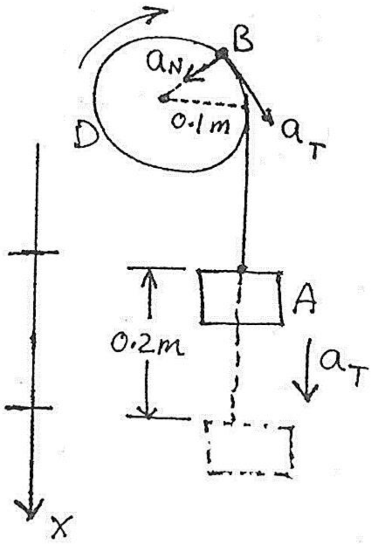
$$a_T = \frac{dv}{dt} = R\,\frac{d\omega}{dt} = R\,\alpha$$

Total Acceleration: $\vec{a} = \vec{a}_N + \vec{a}_T$

Problem 1

A disc D rotating as shown in the figure below. The motion of A is uniformly accelerated, at t=0 the velocity of the body A is 0.04 m/sec and 2 sec later A has fallen 0.2 m. Find the a_T *and* a_N at any time.

Sol.



$$x = v_o t + \frac{1}{2}at^2$$

$$v_o = 0.04\;m/sec$$

$$x = 0.04\,t + \frac{1}{2}at^2$$

At t= 2 sec, x=0.2 m

$$0.2 = 0.04 * 2 + \frac{1}{2} a (2)^2$$

$$a = 0.06\ m/sec^2$$

So, the position of the body at any time is

$$x = 0.04\ t + 0.03\ t^2$$

The velocity of the body at any point

$$v = \frac{dx}{dt} = 0.04 + 0.06\ t$$

$$a_T = \frac{dv}{dt} = 0.06\ m/sec^2$$

$$a_N = \frac{v^2}{R} = \frac{(0.04 + 0.06\ t)^2}{0.1} = 0.016 + 0.048\ t + 0.036\ t^2$$

Force and Motion

Force is any influence which tends to change the motion of an object. There are four fundamental forces in the universe:

1- Gravity force
2- Nuclear weak force
3- Electromagnetic force
4- Nuclear strong force

In mechanics, **forces,** causes the linear motion, whereas **torques,** causes the rotational motion.

The action of forces in causing motion is described by Newton's Laws under ordinary conditions, although there are notable exceptions.

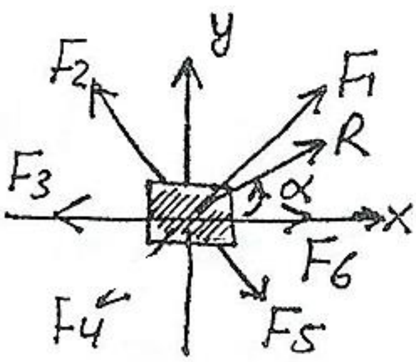
Forces are inherently vector quantities. The SI unit for force is the Newton, where Newton = kg m/s^2.

Composition of Concurrent Forces

If the forces are concurrent (all forces applied at the same point), their resultant is their vector sum. Let the resultant force $\vec{R}$ of several concurrent forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, etc$ is:

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F}_i$$

If the forces are coplanar (x-y): $\vec{R} = iR_x + jR_y$



Where:

$$R_x = F_{1x} + F_{2x} + F_{3x} + \cdots = \sum F_{ix}$$

$$R_y = F_{1y} + F_{2y} + F_{3y} + \cdots = \sum F_{iy}$$

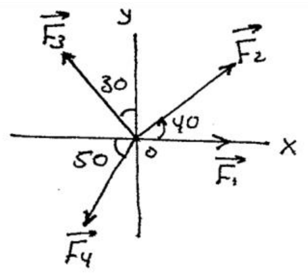
The magnitude of $\vec{R}$ is: $R = \sqrt{R_x^2 + R_y^2}$

The direction of $\vec{R}$ is: $\tan \propto = \frac{R_y}{R_x}$

Problem

Find $\vec{R}$ of the following forces acting on body at 0.

$F_1 = 1200\ N, F_2 = 900N, F_3 = 300\ N, F_4 = 800\ N$



Sol.

$$\vec{F}_1 = 1200\ i$$
$$\vec{F}_2 = (F_2 \cos 40)i + (F_2 \sin 40)j$$
$$\vec{F}_2 = 689.4i + 578.5j$$
$$\vec{F}_3 = (F_3 \cos 120)i + (F_3 \sin 120)j$$
$$\vec{F}_3 = -150i + 259.8j$$
$$\vec{F}_4 = (-F_4 \cos 50)i + (-F_4 \sin 50)j$$
$$\vec{F}_4 = -514.2i - 612.8j$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$
$$R_x = 1200 + 689.4 - 150 - 514.2 = 1225.2\ N$$
$$R_y = 0 + 578.5 + 259.8 - 612.8 = 225.5\ N$$
$$\vec{R} = 1225.2\ i + 225.5\ j$$

$$R = \sqrt{(1225.2)^2 + (225.5)^2} = 1245\ N$$

$$\tan \propto = \frac{R_y}{R_x} = \frac{225.5}{1225.2} \quad \Rightarrow \propto = 10.4°$$

Force and Equilibrium

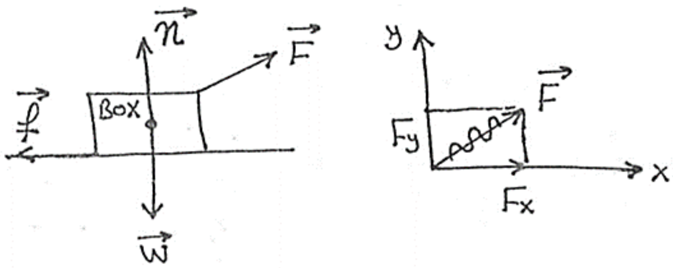
Forces: if unbalanced tend to change the state of motion of a body. A body is said to be in equilibrium if it: 1) is at rest or moves at constant speed in straight line. 2) is either not rotating or is rotating at a constant rate.

These conditions are statements of Newton's First Law of motion.

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F}_i = 0$$

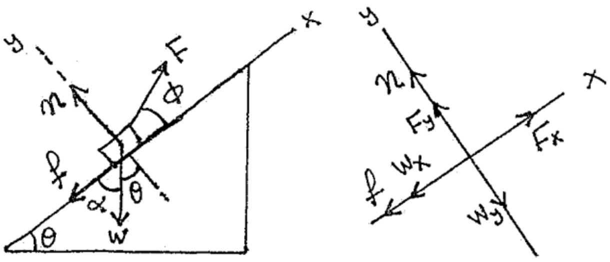
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

In case of Plane surface:



$\vec{F}$: *force of man on box.*
$\vec{n}$: *forceof contact of earth's surface against box.*
$\vec{W}$: *gravetational force of earth on box.*
$\vec{f}$: *friction force of earth against box tending to prevent box from moving to right.*

In case of inclined surface:



$F_x = F\cos\emptyset$ *and* $F_y = F\sin\emptyset$
$W_x = -W\sin\theta$ *and* $W_y = -W\cos\theta$
$n_x = 0$ *and* $n_y = n$
$f_x = -f$ *and* $f_y = 0$

Newton's First Law

Newton's First Law states that: **"An object will remain at rest or in uniform motion in a straight line unless acted upon by an external force"**.

It may be seen as a statement about **inertia**, that objects will remain in their state of motion unless a force acts to change the motion.

Any change in motion involves an acceleration; led to the Newton's Second Law to be applies. In fact, the First Law is just a special case of the Second Law for which the net external force is zero. Newton's First Law contains implications about the fundamental symmetry of the universe.

Newton's Second Law

The second law states that: **"The acceleration of an object is dependent upon two variables: the net force acting upon the object and the mass of the object"**.

The acceleration of an object depends directly upon the net force acting upon the object, and inversely upon the mass of the object. As the force acting upon an object is increased, the acceleration of the object is increased. As the mass of an object is increased, the acceleration of the object is decreased.

$$F_{\text{net external}} = ma$$

Net force on object = mass of object x acceleration

Newton's Second Law applies to a wide range of physical phenomena, but it is not a fundamental principle like the Conservation Laws. It is applicable only if the force is the net external force. It does not apply directly to situations where the mass is changing, either from loss or gain of material, or because the object is traveling close to the speed of light where relativistic effects must be included. It does not apply directly on the very small scale of the atom where quantum mechanics must be used.

Example

Ali's car, which weighs 1,000 kg, is out of gas. Ali is trying to push the car to a gas station, and he makes the car go 0.05 m/s/s. How much force Ali is applying to the car?

$$\mathbf{F = m \cdot a}$$

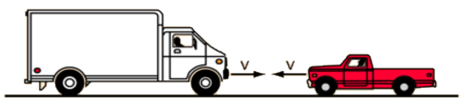
$F=1000 * 0.05 = 50$ N

Newton's Third Law

Newton's third law state that: **“All forces in the universe occur in equal but oppositely directed pairs”**. For every external force that acts on an object there is a force of equal magnitude but opposite direction which acts back on the object which exerted that external force.

In the case of internal forces, a force on one part of a system will be countered by a reaction force on another part of the system so that an isolated system cannot by any means exert a net force on the system as a whole.

Newton's third law is one of the fundamental symmetry principles of the universe. For example, when a small truck collides head-on with a large truck, your intuition might tell you that the force on the small truck is larger. Not so! Comparison of the collision forces for the two trucks: Newton's third law dictates that the forces on the trucks are equal but opposite in direction.

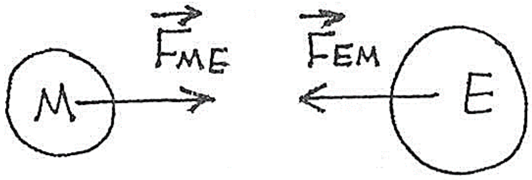


Force	F	=	F
Impulse	F_t	=	F_t
Change in momentum	$m\Delta v$	=	$m\Delta v$
Acceleration	ma	=	ma

Impulse is force multiplied by time, and time of contact is the same for both, so the impulse is the same in magnitude for the two trucks.

Change in momentum is equal to impulse, so changes in momenta are equal. With equal change in momentum and smaller mass, the change in velocity is larger for the smaller truck. Since acceleration is change in velocity over change in time, the acceleration is greater for the smaller truck.

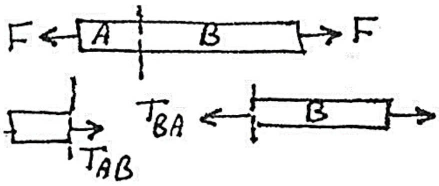
So, Newton’s third law of motion: when two partials interact, the force of one partial is equal and opposite to the force on the other.

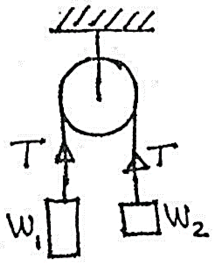


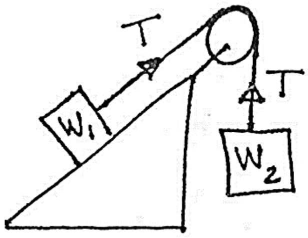
$-\vec{F}_{ME} = \vec{F}_{EM}$ ***Action and Reaction Pairs***

$\grave{F} = -F$ *force of box on man.*
$\grave{n} = -n$ *force of conact of box against earth's surface.*
$\grave{W} = -W$ *gravetational foece of box on earth.*
$\grave{f} = -f$ *force of box on earth tending to drag earth to right.*

Similar things are applied on the tension force







Where T is the tension.

Friction

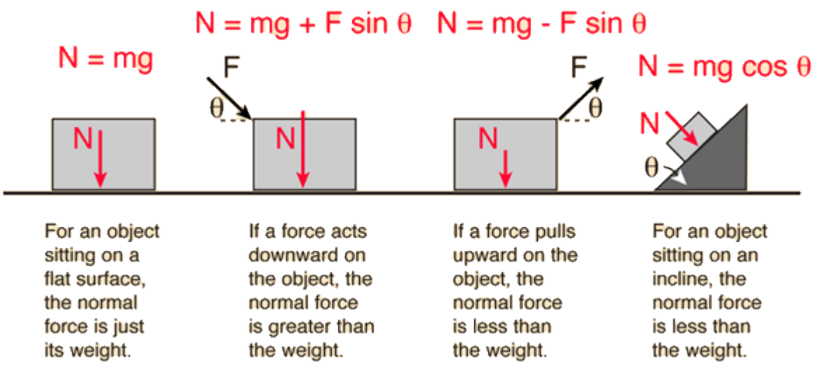
Frictional resistance to the relative motion of two solid objects is usually proportional to **the force which presses the surfaces together** as well as **the roughness of the surfaces**. Since it is the force perpendicular or "normal" to the surfaces which affects the frictional resistance, this force is typically called the **"normal force"** and designated by **N**. The frictional resistance force may then be written:

$$f_{friction} = \mu N$$ **Where μ is the coefficient of friction**

The amount of force required to move an object starting from rest is usually greater than the force required to keep it moving at constant velocity once it is started. Therefore, two coefficients of friction are sometimes quoted for a given pair of surfaces: a coefficient of **static friction (μ_s)** and a coefficient of **kinetic friction (μ_k)**.

Friction is typically characterized by a coefficient of friction which is the ratio of the frictional resistance force to the normal force which presses the surfaces together which acts perpendicular or "normal" to the surfaces. In this case the normal force is the weight of the block. In many common situations, the normal force is just the weight of the object which is sitting on some surface, but if an object is on an incline or has components of applied force perpendicular to the surface, then it is not equal to the weight.

In the figure below the common encountered situations for objects at rest or in straight line motion.



For curved motion, there are cases like a car on a banked curve where the normal force is determined by the dynamics of the situation. In that case, the normal force depends upon the speed of the car as well as the angle of the bank.

The fraction force between two surfaces is parallel to the surfaces in a direction to oppose the relation motion which is occurring (**kinetic fraction**) or the motion which would occur if the fraction were not there (**static fraction**).

1) $Staric\ Fraction \qquad f_s < \mu_s\, n \qquad (\mu_s\ is\ charactestic\ of\ the\ surface)$
2) $Kinetic\ Fraction \qquad f_k = \mu_k\, n$
$(When\ the\ extrenal\ force, tending\ to\ move\ surfaces\ on\ each\ others)$
$Note\ that: \qquad \mu_s > \mu_k \qquad and \qquad \mu < 1$

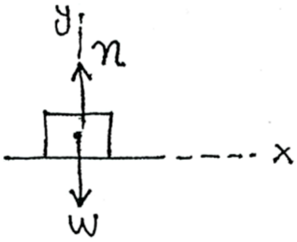
Problem 1
Find all forces acting on the block in the figure below, suppose a smooth fraction less surface.

Sol.

The block in equilibrium $\sum \vec{F} = 0$

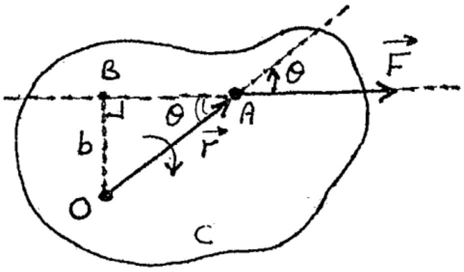
$$\sum F_x = 0$$

$$\sum F_y = n - W = 0$$



Torque

When a force acts on an extended body. The body does not merely move in the direction of the force but usually changes its orientation by turning (rotational motion).



b: Perpendicular distance from acting force line to the point O, also called lever arm: b=OB

Torque = force × lever arm or Torque T=F b (N.m)

The torque is perpendicular on F and r according to the right hand rule.

$\because b = r \sin\theta \quad \Rightarrow T = F\, r \sin\theta \quad \Rightarrow \vec{T} = \vec{r} \times \vec{F}$

Which is vector product (cross product). Where $\vec{r}$ is the position vector.

In x-y plane: $\vec{r} = xi + yj \quad and \quad \vec{F} = F_x i + F_y j$

$$\vec{T} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ x & y & 0 \\ F_x & F_y & 0 \end{vmatrix} = k(x\,F_y - y\,F_x)$$